FQXi Essay Contest 2012: Which of our foundational assumptions are wrong?

Rethinking Einstein's Rotation Analogy

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Abstract Einstein built general relativity (GR) on the foundation of special relativity (SR) with the help of an analogy involving uniformly rotating bodies. Among this analogy's most useful implications are those concerning the need for non-Euclidean geometry. Although GR is well-supported by observations, a curious fact is that almost all of them are of phenomena *over* the surfaces of large gravitating bodies; i.e., they support the *exterior* solution. Whereas the *interior* solution remains untested. In particular, the prediction that the rate of a clock at the center of a gravitating body is a local minimum remains untested. The Newtonian counterpart for this prediction of GR is the common oscillation prediction for a test mass dropped into a hole through a larger gravitating body. The main point in what follows is that this prediction needs to be checked by direct observation. Einstein's analogy serves as a launching pad for bringing out the significance of this experiment as well as exposing possible weaknesses in a few other assumptions, which are then also duly questioned. To facilitate looking upon these problems with fresh eyes, we invoke an imaginary civilization whose members know a lot about rotation but nothing about gravity. Their home is a large and remote rotating body whose mass is too small to make gravity important. What would these people think of Einstein's rotation analogy?

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1 Introduction

> *The treatment of the uniformly rotating rigid body seems to me to be of great importance on account of an extension of the relativity principle to uniformly rotating systems along analogous lines of thought to those that I tried to carry out for uniformly accelerated translation.* [1] — Albert Einstein

When the quote above was first written (1909) it was not yet clear to Einstein that non-Euclidean geometry would be crucial to creating GR. Though uniform rotation eventually helped Einstein to see this, in 1909 it was used in conjunction with another of Einstein's heuristic devices: the equivalence principle (EP). The EP can be thought of as an analogy unto itself, one concerning the relationship between gravity and *acceleration.*

Our (local) experience of gravity at Earth's surface, according to the EP, is as though there were no attractive force of gravity, but instead the ground accelerates upward. This "explains" why all falling bodies appear to fall downward with the same acceleration. As L. C. Epstein playfully stated it: "Einstein's view of gravity is that things don't fall; the floor comes up!" [2]

A naïve interpretation of this description is clearly indefensible for many reasons. Nevertheless, we will not immediately dismiss the whole of it. In the end we will find a not so naïve interpretation; one that appears to be consistent with all known facts and can be easily tested in a large, yet accessible physical domain from which, presently, the facts are lacking. The domain referred to here is the insides of gravitating bodies. Nobody knows what happens to a test object dropped into a hole through a larger body because this experiment has not yet been done. The title and contents of a modern textbook help to explain this situation: *Gravity from the Ground Up.* [3] Nowhere in this 462-page tome are we told of how little we know about *gravity from the ground down*—the most ponderable half of the gravitational Universe.

The problem question motivating this essay clearly implies that we are indeed making at least one wrong foundational assumption. Echoes of this assessment are not uncommon these days. An effective strategy for trying to solve such a stubborn puzzle is to employ any mind loosening, perspective shifting method that might facilitate finding the solution. Therefore we will later try seeing through the eyes of an imaginary civilization that has no knowledge of gravity at all. Before considering this alien perspective, we first consider Einstein's perspective. Before closing we will find that the perspective of our "gravity virgins," as it were, will evoke a range of foundational questions significantly beyond what they see as the *cause* for the physical manifestation of non-Euclidean geometry.

2 Einstein's Guiding Principles

Uniform rotation and gravitation both coexist with four key physical effects: 1) acceleration, 2) velocity, 3) length contraction, and 4) time dilation. Suppose that r in both cases represents the distance from the center; ω is the angular velocity, G is Newton's constant, and M is the mass of the gravitating body. We may then identify key differences between our analogous causes. In the case of rotation the acceleration and the velocity ($r\omega^2$ and $r\omega$, respectively) clearly apply to the rotating body itself. Whereas in the case of gravity the acceleration and velocity (GM/r^2 and $\sqrt{2GM/r}$) are supposed to apply to *falling* bodies. The gravitational velocity is, moreover, only that of one special case: an object fallen from infinity.

Two other key differences are that, 1) the accelerations (as measured by an accelerometer) are opposite in direction. On the rotating body it is inward; on the gravitating body it is outward. And 2) the rotational velocity is tangential, whereas the gravitational velocity is radial. The direction of the velocity bears especially on the effect on rod lengths. On the rotating body rods are shortened tangentially, whereas on the gravitating body they are shortened radially. The effect on clock rate, by contrast, does not depend on direction, only on magnitude.

Now let's consider what Einstein makes of these facts. Recall that our opening quote is from 1909, when Einstein perceived how uniform rotation relates to the EP, but not yet to non-Euclidean geometry. The latter connection did not occur to Einstein till 1912. An example from 1914, which still emphasizes the EP connection is given here first:

> The following important argument speaks for the relativistic perspective. The centrifugal force that works on a body [that is part of a rotating system K'] under given conditions is determined by precisely the same natural constants [i.e., its mass] as the action of a gravitational field on the same body, in such a way, that *we have no means to differentiate a 'centrifugal field' from a gravitational field.* . . . This quite substantiates the view that *we may regard the rotating system K' as at rest* and the centrifugal field as a gravitational field. [4] [Emphasis added.]

A second example reinforces the point. In his book on SR and GR intended for lay readers Einstein discusses the observations made by an observer who is riding along with the rods and clocks attached to a rotating disk:

> The observer on the disc may regard his disc as a reference-body which is "at rest"; on the basis of the general principle of relativity he is justified in doing this. The force acting on himself, and in fact on all other bodies which are at rest relative to the disc, he regards as the effect of a gravitational field. . . . This gravitational field is of a kind that would not be possible on Newton's theory of gravitation. But since the observer believes in the general theory of relativity, this does not disturb him.[5]

Following this latter example, Einstein explained that various experiments with rods and clocks carried out by the rotating observers would lead to the conclusion that Euclidean geometry no longer corresponds to experience. For example, the circumference of the disk no longer equals $2\pi r$.

It is not just non-Euclidean geometry that Einstein appeals to in the context of these quotes. He also appeals to a variety of "principles." Explicitly mentioned above are the *relativity principle*, the *principle of general relativity*, and the *EP*. We see in Einstein's writing that he also sought to establish agreement with *Mach's principle* and the *principle of general covariance*. For good measure, we should add the *principle of local Lorentz invariance*.

We will not be concerned with the details of these principles or the (still unsettled) questions as to exactly how well they were satisfied or how meaningful they are. Rather we simply note what a very "principled" theorist Einstein was. We note also what an extreme position he tried to defend: A rotating body can be regarded as being *at rest?!* This means that a rotating observer must regard the whole Universe as revolving around himself. Is this not a rather bizarre and solipsistic idea?

As beneficiaries of all that is correct about Einstein's final mathematical theory, we may still find its curious origins a little hard to believe. Finding them even harder to believe would be an imaginary civilization who were not only deprived of Einstein's principles and theories, but who evolved without any understanding of gravity.

3 Motion-Sensing Rotonians

How we conceive the physical world depends a lot on our environment. Perhaps the perspective that has evolved from the surface of our warm, moist 5.97×10^{24} kg sphere (Earth) is flawed in a way that would appear obvious to beings who live in a wholly different environment. Let us therefore imagine a civilization that evolved in a large rotating wheel far removed from any bodies massive enough to produce an appreciable gravitational field. Suppose the wheel's size is that of a large Earthian city; it is called *Roton* and its inhabitants are *Rotonians*. Even though they have no conception of gravity, the society of Roton is otherwise advanced. Their scientific instruments are extremely reliable and capable of measuring small changes in distance, velocity, acceleration, and clock frequency. Also Rotonian mathematicians are well versed in non-Euclidean and higher dimensional geometries.

Presently the Rotonians are planning an excursion to solve the mystery of the distant myriad points of light that have filled their dreams for as long as they can remember. Before chronicling their adventure, let's take stock of their understanding of motion. Aside from telescopes, gyroscopes and various complex inertial guidance systems, two of their most basic motion-sensing devices are accelerometers and clocks. Accelerometers stationed at various distances from the rim of Roton inform them of the force experienced by bodies at these positions. When exploring their exterior neighborhood accelerometers serve just as reliably to indicate the propulsive force of their rockets. A key fact ingrained in all Rotonians is that accelerometers are utterly reliable gauges of acceleration. If an accelerometer gives a non-zero reading it means the device is being forced to move in the indicated direction with the indicated magnitude. Especially noteworthy is that, to a Rotonian, if an accelerometer reading is *zero*, the instrument is certainly *not accelerating.*

Rotonians understand clocks to be motion sensing devices unto themselves because of their change in frequency due to velocity. Presently we consider their role as components of more complex and sophisticated communications and positioning systems. Roton is equipped with an array of synchronized clocks and electromagnetic wave relays. The most sensible method of clock synchronization, which they have adopted, is by way of a signal from the axis. Early in the development of this system the Rotonians discovered a crucial asymmetry. The speed of light in one direction of motion is faster than the constant, c, and in the opposite direction it is slower than c. To first order, the speed differences are equal to the speed of rotation, $\pm\, r\omega$. Since some of their communication and positioning needs are most demanding, if this speed asymmetry is not taken into account serious accidents could occur. Even when the rod-measurable distance is the same in opposite directions, transmitting *with* the rotation direction is slower than transmitting *against* it.

Earthian readers who may have the impression that the speed of light is always the same in any direction should bear in mind that the synchronization method of our Global Positioning System is essentially the same as the Rotonian one, for the same reason. It takes account of the asymmetry in light speed due to Earth's rotation. Havoc would prevail upon Earth if we had fancied to "synchronize" clocks by the Einsteinian prescription, one-by-one around the circumference. More sophisticated Earthian readers may think, yes this is true, but the speed of light is at least "locally" equal to c for all observers. As it turns out, the Rotonians never got the memo giving the order to obey this principle (local Lorentz invariance); nor have they ever suffered for it. So we'll continue with their story.

For many years the Rotonians have had their array of clocks in place. It includes one at the axis, many along the rim, and many in between. Careful observation of these clocks has provided the Rotonians with ample evidence of the effect of velocity on clock frequency. Recognition of clocks as motion-sensing devices has proven to be true not only for clocks rigidly stationed on the structure of Roton, but also in the (somewhat trickier) case of linear velocity, as when they venture beyond Roton's confines. For the sake of brevity we need to leave out any additional subtleties in the Rotonian understanding of motion, light, space and time. Suffice it to say that accelerometers and clocks will play the most significant role as motion-sensing devices in what follows.

The day finally comes for our intrepid explorers to take off to parts unknown. Fast forward many years: The Rotonians awaken from their pre-arranged statis to find themselves nearing what they eventually learn is a "planet" called Earth. What a bewildering experience! This colossal ball of matter appears to be accelerating toward them with ever increasing magnitude. In the nick of time they turn around and blast their rockets so as to accelerate upwardly and make a soft landing. Imagine their astonishment when the Rotonians learn that the acceleration of the planet toward them would have been the same from any angle of approach. Accelerometers all the way around the globe say that its surface is "coming up." From their Earthian hosts, they learn that this effect is called gravity. What they do not understand is that the natives think of the planet as being static. Most Earthians say that a "falling" accelerometer, whose reading is zero, is actually accelerating downward. This sounds like utter nonsense to the Rotonians.

Nor does the impression of nonsense dissipate when told by a certain faction of Earthians that positive accelerometer readings do indeed mean the ground is accelerating upward, because even this faction still regards the Earth as a whole as *static*. Earthians, the Rotonians surmise, are schizoid.

Being compassionate scientists, the Rotonians are eager to gather evidence to settle the matter. The stakes are clearly high, as it is in the blood of any Rotonian to regard a non-zero accelerometer reading as indicating only one or a combination of two things: 1) rotation and/or 2) a source of propulsion. Now they need to admit the possibility—depending on the results of their investigation—that they must add: 3) a state of *rest* in a gravitational field. Since accelerometer readings on Earth's surface are positive in the outward direction, the dominant effect of the planet is obviously not rotation.

Rotonians suspect that the crucial evidence lies *inside* the planet, or in prinicple, inside any body of matter—under the hood, so to speak. In accordance with the possibilities mentioned above, they instinctively suspect that matter is a source of propulsion. This possibility did not occur to them earlier because they had not before encountered such a big chunk of it. In terms of their unforgettable landing experience, the Rotonians want to determine what would have happened if, instead of landing on the surface, their approach took them into an evacuated hole through the planet, which allowed them to fall (rockets off) as far as they would, toward the center. Obviously this can't be done with the planet itself, but it could be done in a laboratory with much smaller bodies.

From the Earthian theory of gravity Rotonians have learned of the oscillation prediction for their experiment. If this prediction is correct it would substantiate (3) above. Their own prediction, which is based on their understanding of accelerometer readings, is that the test object will not pass the center. To them it is obvious that nothing ever pulls the test object downward. The Rotonians are a little surprised that Earthians had not thought of testing their oscillation prediction before, but they are delighted to bring the possibility to their attention. Happily, the mere presence of the friendly alien Rotonians has induced a new sense of mental flexibility amongst the Earthians. With abundant enthusiasm Earthian scientists join the Rotonians in their experimental pursuit.

4 Curvature Coefficients

As plans to do the experiment get underway, Rotonian theorists eagerly absorb all they can about the Earthian ideas of gravity and motion. They especially seek out what is common to both worlds. Of special importance is their discovery in Earthian archives of a mathematical expression for the fact that the speed of light is a physical limit. From their own experiments with light and clocks and the effect of velocity, the Rotonians had long ago derived an identical equation. Its meaning is as follows. If a rocket is provided with a huge fuel supply allowing it to maintain the same acceleration for a very long time, its velocity will continually increase, but it cannot reach the speed of light. The equation is

$$v = \frac{at}{\sqrt{1 + a^2t^2/c^2}} \ , \tag{1}$$

where a is the acceleration as indicated by an onboard accelerometer and t is the time given by a clock in the original reference frame. We'll return to this equation below.

Unknown to the Rotonians before their trip to Earth is the significance of Newton's constant G. To make concrete sense of it they imagine the Earth as having planted upon its surface an array of extremely tall towers extending vertically many diameters into space. From the Earthian theory, they find especially significant that, if they had never fired their rockets for a soft landing—i.e., if they had only just "fallen" to Earth (and neglecting the effect of the Sun and other planets) with rockets off, then the relative speed between points along one of these towers and their rocket would have been $\approx \sqrt{2GM/r}$. Corresponding to this speed is the acceleration $g \approx GM/r^2$. Accelerometers placed at various locations along the tower confirm these accelerations. And clocks fastened alongside the accelerometers are found to have frequencies that vary as

$$f(r) \approx f_0\sqrt{1 - \frac{2GM}{rc^2}}\,, \tag{2}$$

where f_0 is the rate of a clock at infinity. Note that GR predicts this equation to be exact, not just approximate. Whereas the hypothesis the Rotonians are beginning to develop leads them to a different exact expression, as we will see later. For most circumstances the differences are extremely small. Eq. 2 is clearly analogous to the corresponding equation involving rotation:

$$f(r) = f_0\sqrt{1 - \frac{r^2\omega^2}{c^2}}\,. \tag{3}$$

In both cases the rate of a clock has a well-defined dependence on radial distance just as velocity has a well-defined dependence on radial distance.

The above facts and equations are among the clues that Rotonians have gathered about gravity. We will see that Eq. 1 plays a key role in tying together the various puzzle pieces of gravity into a coherent picture. To make better sense of the Rotonian reasoning, we need first to digress—to widen our context and to address a few objections.

One of the foremost objections, I suspect, concerns the energy conservation law. The Rotonians are well aware that their prediction for the interior radial falling experiment, if confirmed, would violate this law. The most immediate defense is simply that this law has not yet been tested inside matter. Is this not all the more reason to do the experiment? The Rotonians' working hypothesis, in a nutshell, is that matter is a source of propulsion and the source of space. As far as they can tell, this is what gravity is. The hypothesis goes quite naturally with the idea that energy perpetually increases.

Perceiving how radical such a notion is, the Rotonians understand that it could not possibly be true if there were only three dimensions of space. The basis of the Rotonian scheme is that accelerometers always tell the truth. But this makes no sense for acceleration along their rigid towers, for example, if one tries to naïvely conceive the acceleration as an expansion in 3D space. The acceleration at r is four times what it is at $2r$. In 3D space the indicated accelerations would cause the system to disintegrate.

Enter a fourth spatial dimension and the conception that accelerometers tell the truth becomes a logical possibility. We can here provide only a basic sketch of this deep subject. An "extra" dimension means having a new direction in which to move. A lower dimensional analogy helps to see this. Imaginary creatures who live on a "2D" spherical surface can deduce the existence of a higher dimension by a combination of facts: The angles of triangles drawn on the surface do not add up to 180°; and a straight line path returns to its starting point.

The key here is the relationship between *curvature* and *higher dimensions.* As we have just seen, if a space of seemingly lower dimension is demonstrably curved, then in fact there exists one higher dimension *for the space to curve into.* A curved *line* defines a 2D plane, a curved *surface* defines a 3D volume, etc. Mathematicians and general relativists may argue that it is not necessarily so. But it's a lot easier to see how it really does seem to be so. Taking the easy route we conclude as follows. Because our seemingly 3D world [or (3+1)-D world, including time] is known to exhibit spacetime curvature, this *means* that it is embedded in a higher dimension. Such a world is actually (4+1)-dimensional. Furthermore, we now have an idea *why* it is curved. Spacetime is curved because it *moves.* For both rotating bodies and gravitating bodies *the cause of spacetime curvature is motion.*

This can be partially represented graphically as in Figure 1. The abscissa is radial distance from the center of a gravitating body. The ordinate has the dual function of velocity and—because we imagine the whole graph as the cross-section of a flaired rotating tube—extension into the fourth spatial dimension. The outer envelope thus corresponds to points along the Rotonian towers. The whole system is in a state of *stationary motion* (analogous to a rotating body). It appears rigidly coherent, yet different radial distances have different accelerations and velocities. If all bodies of matter are plotted on identical graphs, then the idea is that they all have the same rotational frequency, whose magnitude is determined by Newton's G. This "tubular rotation" is $(4 + 1)$-dimensional. We deduce its reality from our motion-sensing devices.

The waveforms drawn on the graph are to be thought of as helices drawn around the whole tubular surface. Notice that their pitch angles are 45°. This means that their rotational speeds everywhere equal the apparent speed of the projected intersection along the axis. Outside the gravitating body ($r > R$) the apparent acceleration of these intersection points is everywhere $\approx GM/r^2$. A discussion of the motion inside the body ($r < R$) will be given elsewhere.

With the above ideas in mind, we can now begin to understand the key distinctions the Rotonians make between Eq. 2 and Eq. 3. Equation 3 represents the effect

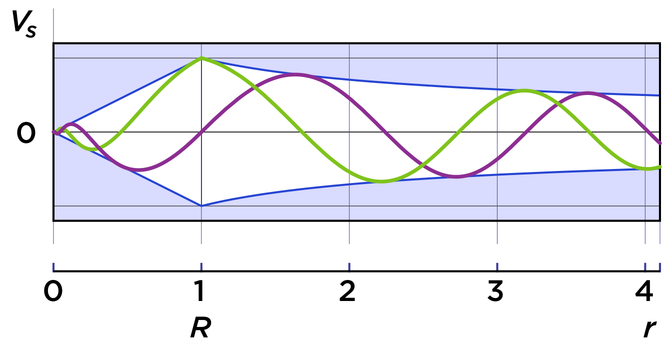

Fig. 1 Inspired by L. C. Epstein's tubular modeling of GR, [6] the Rotonian tubular model—whose cross-section is shown here—differs markedly, first of all, because it *moves*. Epstein's model was designed to accommodate the static block time picture and to represent motion *through* space, as frozen paths on a curled up spacetime diagram. The Rotonian model represents motion *of* space—into or outfrom a fourth spatial dimension. (See text.)

on the rate of a clock due to rotational motion *through* space. Whereas the corrected version of Eq. 2 (see below) represents the effect on the rate of a clock due to gravitational motion *of* space. Eq. 1 clearly also represents motion through space—not rotational, but linear. In Eq. 1 light speed c is approached with the increase of *time*. The Rotonians suppose that the motion of space should be similarly limited. So they propose the modified expression which replaces the linear motion (at) with the gravitational motion $\sqrt{2GM/r}$:

$$V_\mathrm{S} = \frac{\sqrt{\frac{2GM}{r}}}{\sqrt{1+\frac{2GM}{rc^2}}} = \sqrt{\frac{2GM}{r+\frac{2GM}{c^2}}}\,. \tag{4}$$

In this equation light speed is approached with the increase of M and/or with a decrease in r. Since it applies to all points of a sphere at coordinate distance r surrounding the mass M, Rotonians interpret Eq. 4 as representing a volumetric [(4+1)-dimensional] motion of space. This idea corresponds to the idea of Figure 1. Squaring both sides we get,

$$V_\mathrm{S}^2 = \frac{2GM}{r(1+\frac{2GM}{rc^2})} = \frac{2GM}{(r+\frac{2GM}{c^2})}\,. \tag{5}$$

The length in the denominator on the right side is the sum of the *coordinate radius*, r and the *gravitational radius*, $2GM/c^2$. This suggests that, whatever the coordinate radius may be, by virtue of its mass, a body possesses an additional spatial extent. Motivated by the suggestiveness of Eq. 4 and Eq. 5, the Rotonians propose a divergence from standard GR by treating $2GM/rc^2$ as a quantity to be *added* to rather than *subtracted* from unity. They assume that the quantity $(1 + 2GM/rc^2)$ appearing in (5) plays a role similar to the metric coefficient $(1 - 2GM/rc^2)^{-1}$ appearing in the Schwarzschild solution—applying to both space and (its inverse) to time. Based on these arguments we see that the Rotonian correction to Eq. 2 is:

$$f(r) = \frac{f_0}{\sqrt{1+2GM/rc^2}}\,, \tag{6}$$

5 Theoretical and Astrophysical Consequences

The first consequence discussed in this section is one of agreement with standard predictions rather than conflict. As shown in more detail in [7], when the coefficient in Eq. 5 is used to derive an equation for the acceleration due to gravity, we find a maximum that depends only on mass, not coordinate radius: $(g_{max} = c^4/4GM)$. From this equation follows another which gives a *maximum force.* Curiously, this agrees exactly with a tentative "proposal" based on GR derived by G. W. Gibbons [8], and an explicit prediction, by C. Schiller. [9] They both found $F_{max} = c^4/4G$, as do we, by our simple analysis beginning with Eq. 1. The noteworthy difference is that Gibbons and Schiller maintain the correctness of GR, with its predicted horizons and singularities. Whereas our result is horizon and singularity-free.

Another key consequence, which is also discussed in more detail in [7], is the relationship between the Rotonian experiment proposal and the way clock rates vary inside matter. The standard Schwarzschild interior solution predicts that the rate of a clock at the center of a massive sphere would be a local *minimum.* This doesn't make sense to the Rotonians because there is no tangible evidence of motion at the center. Instead, they see the sphere's center as being analogous to a rotation axis, at which clock rates would be a local maximum, not minimum.

This prediction of a local maximum clock rate at $r = 0$ (the derivation of which we omit for brevity) goes with the non-oscillation prediction for the experiment. It also goes with the existence of dark dense astronomical bodies that, from a distance should be rather hard to distinguish from bodies that are customarily regarded as black holes.

6 Conclusions

By rethinking Einstein's rotation analogy we expose much that needs to be questioned in the foundations of physics. If the "Rotonian" hypothesis is supported by experiment, then a wholesale replacement of cherished principles would be in order. The consequences for energy, space, time, and mass are as follows:

1. Energy conservation would be violated. The non-oscillation prediction entails that the energy of matter continually increases as matter perpetually generates space.
2. The inhomogeneous pattern of stationary outward motion (generation of space) is not logically possible if there are only three spatial dimensions. A fourth infinitely large spatial dimension is indicated.
3. The perpetual generation of space and increase in energy solves the problem of time. The interior radial falling experiment may be the ideal testing ground for time reversal invariance as well as the concept of *block time*. If the test object oscillates as per Newton and Einstein, then an ideal video of the motion looks the same whether it is played forward or backward. Whereas if the non-oscillation prediction is confirmed, the video must be played forward for events to unfold in the proper physical order. Time only increases because space and matter also only increase.
4. The nature of mass is revealed as a very simple thing. A body of matter resists being accelerated in any *one* direction (inertia) because—and depending on how much—it is generating space in *every* direction (gravity).

It is hereby proposed that matter, space, and time are interdependent elements in a perpetual process of motion into (or outfrom) the fourth spatial dimension. All of which suggests that the most important presently unanswered foundational question may be: To oscillate or not to oscillate? (See Figure 2.)

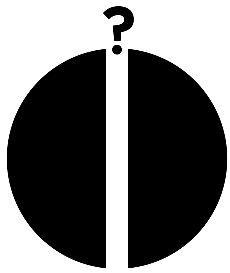

Fig. 2 SLENC — Small Low-Energy Non-Collider. For a more suitable laboratory setup, see [10].

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